DEFORMATION ANALYSIS OF AN INFLATED CYLINDRICAL MEMBRANE BY FINITE ELEMENT METHOD

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1. INTRODUCTION

The main purpose of this paper is the simulation of the inflated composite cylindrical membrane made of rubber matrix reinforced by textile cords. Some orthotropic and transversely hyperelastic constitutive models appropriate for such type of material can be found in literature. Most of them are represented by strain energy function in the form of a polynomial, exponential or logarithmic [9, 10] function of strain invariants regarding the assumed material orthotropy. However the development of the constitutive theory of anisotropic elastic or viscoelastic materials at finite strains is still far to be complete and the publications in this field are sparse. The constitutive equations of the transversally isotropic material in the nonlinear stress and deformation domain are presented in the papers of Holzapfel and coll. [9], Bonet and Burton [8] and Verron [4].

The consistent constitutive model of direction dependent hyperelastic material presented in papers of Ogden, Holzapfel, Gasser and coll. [9] is applied by authors to the problem of the mechanical response of arterial walls and of fiber reinforced composites at finite strains.

In this pursuit the material parameters of strain energy function were identified. Axisymmetric finite elements with the hyperelastic orthotropic material model were made up in Matlab to calculate the deformation of an inflated cylindrical membrane of composite with rubber matrix reinforced by textile cords. The deformation of an inflated cylindrical membrane was compared with the solution by FEM in Ansys.

2. IDENTIFICATION OF MATERIAL PARAMETERS

Assume the isochoric deformation and neglect the dissipation due to irreversible effects. The energy stored in the fibers is assumed to be governed by an exponential function. The free energy function in two dimensional problem can be supposed in the form [1, 2]

$$\Psi(\lambda_1,\lambda_2)=\sum_{i=1}^{3}\frac{\mu_i}{\alpha_i}\left(\lambda_1^{\alpha_i}+\lambda_2^{\alpha_i}+\lambda_1^{-\alpha_i}\lambda_2^{-\alpha_i}-3\right)+\frac{k_1}{k_2}\left\{\exp[k_2(\lambda_2^2\cos^2\alpha+\lambda_1^2\sin^2\alpha-1)^2]-1\right\}, \quad (1)$$

λ_1 and λ_2 are the axial and circumferential stretches respectively, and α is the angle of the two families of reinforcing fibers. We suppose the reinforcing fibers are double-helically arranged in the matrix material symmetrically to the circumferential direction. The angle α of fibers is supposed to be 52^o. The parameters μ_i and α_i are material constants from Ogden’s model of rubber [9]

$\mu_1 = 630$ kPa, $\mu_2 = 1.2$ kPa, $\mu_3 = -10$ kPa, $\alpha_1 = 1.3$, $\alpha_2 = 5$, $\alpha_3 = -2$,

The stress-like parameter k_1 and the non-dimensional parameter k_2 were determined from the experimental measurement of the deformation of membrane and from the 2D cylindrical membrane approximation [1-2].

The Cauchy stresses are defined as the partial derivatives of strain energy function Ψ with respect to the deformation [9, 10]. We have the following expressions:

$$\sigma_1 - \sigma_3 = \lambda_1 \frac{\partial \Psi(\lambda_1, \lambda_2)}{\partial \lambda_1}, \qquad \sigma_2 - \sigma_3 = \lambda_2 \frac{\partial \Psi(\lambda_1, \lambda_2)}{\partial \lambda_2}, \tag{2}$$

The theory of nonlinear membranes has been presented by Green and Adkins [1-2] and applied to various inflated structures [10]. The quasi-static equilibrium equations of problem are

$$\frac{d}{ds}(T_1 r) = T_2 \frac{dr}{ds}, \qquad \kappa_1 T_1 + \kappa_2 T_2 = p, \quad T_1 = \mathrm{h}\sigma_1, \quad T_2 = \mathrm{h}\sigma_2, \tag{3}$$

where p is the inner pressure, T_1 and T_2 are the stress resultant forces per unit length in the meridional and circumferential directions, h is the thickness in the deformed configuration. The Cauchy stress is determined from the equilibrium.

After the substitutions into equations (2) we obtain set of the nonlinear equations for the two variables k_1 and k_2. The determined values were k_1=4.187e+04 kPa and k_2= -23.775. The function of the Helmholtz energy potential for these parameters is convex [2].

3. DETERMINATION OF DEFORMED PROFILES OF MEMBRANE BY FEM

3.1 The axisymmetric membrane element

The element we used here is similar to the element in the paper of Shi and Moita [7], where total Lagrangian approach is adopted (Fig 1). Based on isoparametric formulation, the two displacements U_R and U_Z and the radial coordinate R in the undeformed configuration are interpolated by linear function:

$$\mathrm{U}(\xi) = \begin{Bmatrix} \mathrm{U_R}(\xi) \\ \mathrm{U_Z}(\xi) \end{Bmatrix} = \mathrm{N}(\xi)\left\{\mathrm{U}_{R_1} \quad \mathrm{U}_{Z_1} \quad \mathrm{U}_{R_2} \quad \mathrm{U}_{Z_2}\right\}^{\mathrm{T}}; \quad \mathrm{R}(\xi) = \mathrm{N}(\xi)\left\{\mathrm{R}_1 \quad \mathrm{Z}_1 \quad \mathrm{R}_2 \quad \mathrm{Z}_2\right\}^{\mathrm{T}} \tag{4}$$

$\mathrm{N}(\xi) = \frac{1}{2}\begin{Bmatrix} 1-\xi & 0 & 1+\xi & 0 \\ 0 & 1-\xi & 0 & 1+\xi \end{Bmatrix}$ is the shape function

3.2 Principle of Virtual Work

Neglecting inertia and body forces, the Principle of Virtual Work can be written in the following form [5]:

$$\mathrm{g}(\mathbf{u}, \delta\mathbf{u}, \mathrm{p}) = \int_{\Omega_0} \delta W dV - \int_{\partial\Omega} \delta\mathbf{u} p \mathbf{n} dS = 0 \tag{5}$$

in which $\delta\mathbf{u}$ stands for a virtual displacement vector and $\partial\Omega$ represents the deformed membrane surface.

The Principle of Virtual Work can be expressed as

$$\mathrm{g}(\mathbf{u}, \delta\mathbf{u}, \mathrm{p}) = \delta\mathbf{U}^{\mathrm{T}}\, \mathbf{G}(\mathbf{U}, \mathrm{p}) = 0 \tag{6}$$

In this equation **U** is the nodal displacement vector and **G**(**U**,p) is the

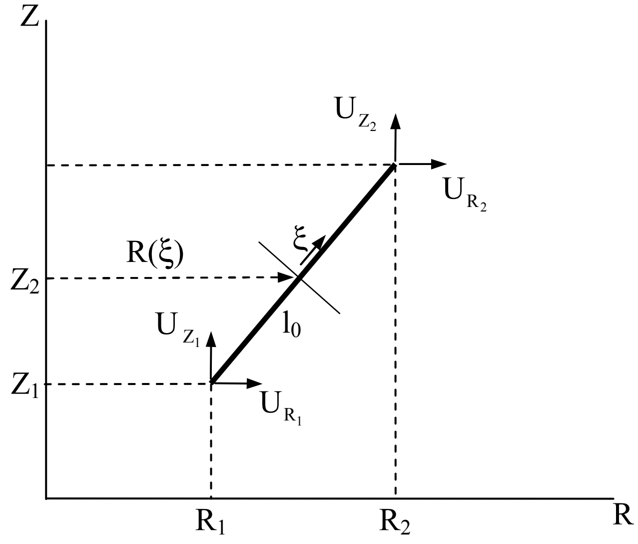

Fig. 1 An axisymmetric membrane element

out-of-balance force which must be equal to zero to ensure equilibrium and can be written under the following form

$$\mathbf{G}(\mathbf{U}, p) = \mathbf{f}_{int}(\mathbf{U}) - p.\mathbf{f}_{ext}(\mathbf{U}) \tag{7}$$

where $\mathbf{f}_{int}(\mathbf{U})$ and $\mathbf{f}_{ext}(\mathbf{U})$ are the internal and external forces, respectively.

It is obvious that the system (7) is highly non-linear both geometrically and by the constitutive equation. Therefore, the classical tangent stiffness **K** has to be defined in order to solve the problem :

$$\mathbf{K} = \mathbf{K}_{int} - \mathbf{K}_{ext} = \frac{\partial \mathbf{f}_{int}}{\partial \mathbf{U}} - p\frac{\partial \mathbf{f}_{ext}}{\partial \mathbf{U}} \tag{8}$$

In this equation the internal stiffness matrix K_{int} is the sum of two terms:

$$\mathbf{K}_{int} = \mathbf{K}^{I}_{int} + \mathbf{K}^{II}_{int} \tag{9}$$

As mentioned above, the problem is highly non-linear. A very powerful to handle this problem is to use the Newton-Raphson iterative method [5-7]. We applied this method to solve the problem of inflated cylindrical membrane. The deformed profiles of the membrane at different stages of loading are on the figure 2

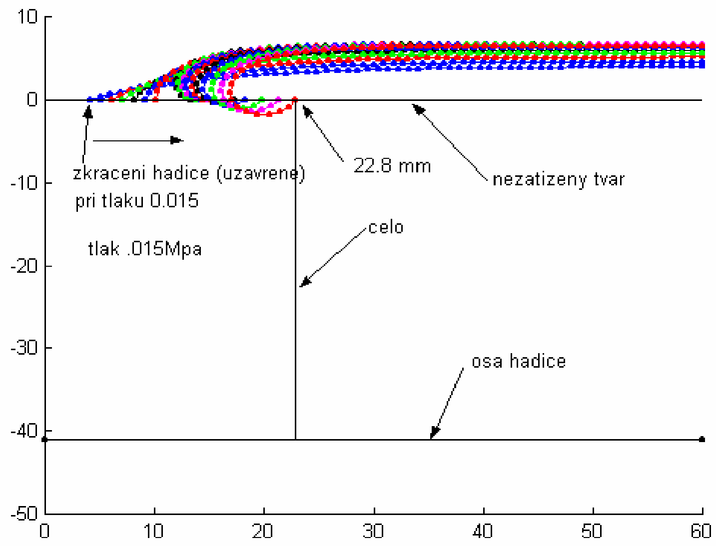

Fig 2. The deformed profiles of the membrane at different stages of loading

4. SIMULATION OF INFLATED CYLINDRICAL MEMBRANE IN ANSYS

In the version Ansys 6.0 is not implemented material model for anisotropic hyperelastic material. Consequently we simulated approximately inflated membrane as isotropic material and using Ogden‘s model. The deformed profile of inflated membrane at pressure 0.01MPa is given in figure 3.

5. CONCLUSIONS

The problem of the identification of the material parameters was solved. The proposed strain energy function was implemented into the calculus of deformations of the cylindrical membrane of air-spring. The deformed profiles of inflated cylindrical composite membrane were determined by FEM in Matlab.

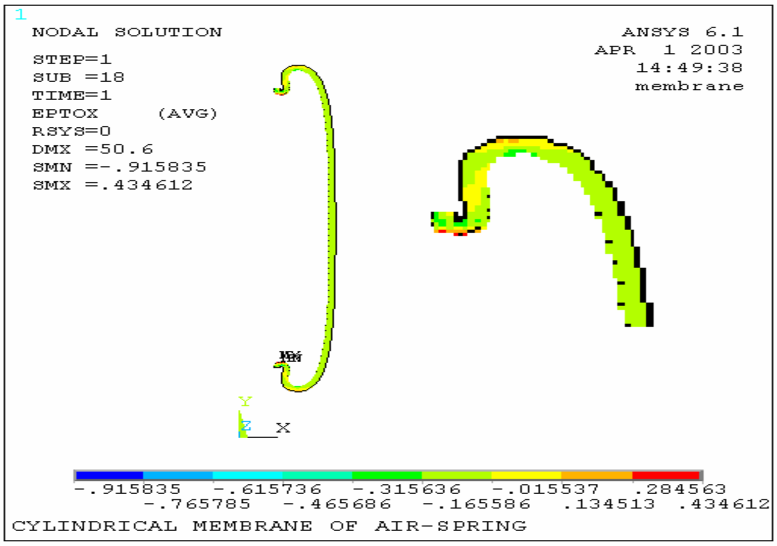

Fig 3. The deformed profile of the membrane

6. ACKNOWLEDGEMENT

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