IDENTIFICATION OF MATERIAL PARAMETERS AND DEFORMATION ANALYSIS OF AN INFLATED CYLINDRICAL MEMBRANE OF COMPOSITE WITH RUBBER MATRIX REINFORCED BY TEXTILE CORDS

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In this paper we present an attempt to identify experimentally the coefficients of strain energy function of the hyperelastic orthotropic material of the thin cylindrical air-spring. The components of the deformation gradient are determined from measured displacements of the grid points drawn on the cylindrical surface of the spring. The true Cauchy stress tensor is calculated from the membrane theory. The deformed shape of the spring surface is determined from the photographic records. The strain energy function is expressed in terms of tensorial invariants with regard to the assumed material symmetry. The coefficients are determined by means of the nonlinear least squares method. The deformation field is then calculated by solving the system of five first-order ordinary differential equations with the material constitutive law and proper boundary conditions.

1. Introduction

The development of the constitutive theory of anisotropic elastic or viscoelastic materials at finite strains is still far to be complete and the publications in this field are sparse. The constitutive equations of the transversally isotropic material in the nonlinear stress and deformation domain are presented in the papers of Verron and coll. [1], Bonet and Burton [2]. We use the consistent constitutive model of direction dependent hyperelastic material presented in papers of Ogden, Holzapfel, Gasser and coll. [3] applied by authors to the problem of the mechanical response of arterial walls and of fiber reinforced composites at finite strains. They use a particular Helmholtz free energy function, which allows modeling the behavior of the orthotropic composite in large nonlinear deformations. The deformation field is generally determined by the finite element method, however, we use the method of the numerical integration of the system of the ordinary differential equations of problem described for isotropic membrane by Guo [4]. We incorporated into this procedure our own orthotropic material law. This method appeared to be quite promising and we presume to use it for the inverse identification of material parameters. Details on the experimental setup and the experiment evaluations can be found in the previous papers of authors [5-7].

2. Determination of material parameters

We assume the isochoric deformation and we neglect the dissipation due to irreversible effects. The energy stored in the fibers is assumed to be governed by an exponential function. The free energy function in two dimensional problem can be supposed in the form [3]

$$W(\lambda_1,\lambda_2)=\sum_{i=1}^{3}\frac{\mu_i}{\alpha_i}\left(\lambda_1^{\alpha_i}+\lambda_2^{\alpha_i}+\lambda_1^{-\alpha_i}\lambda_2^{-\alpha_i}-3\right)+\frac{k_1}{k_2}\left\{\exp[k_2(\lambda_2^2\cos^2\alpha+\lambda_1^2\sin^2\alpha-1)^2]-1\right\}, \quad (1)$$

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λ_1 and λ_2 are the axial and circumferential stretches respectively, and α is the angle of the two families of reinforcing fibers. We suppose the reinforcing fibers are double-helically arranged in the matrix material symmetrically to the circumferential direction. The angle α of fibers is supposed to be 52^o. The parameters μ_i and α_i are taken from Ogden's model of rubber [3] as

μ_1 = 630 kPa, μ_2 = 1.2 kPa, μ_3 = -10 kPa, $\alpha_1 = 1.3$, $\alpha_2 = 5$, $\alpha_3 = -2$,

The stress-like parameter k_1 and the non-dimensional parameter k_2 were determined from the experimental results and from the 2D cylindrical membrane approximation. The equilibrium relations for the infinitesimally long cylindrical membrane in the deformed configuration are:

$$\sigma_1 - \frac{r_i^2}{2hr} p = 0 \ ; \quad \sigma_2 - \frac{r_i}{h} p = 0 \tag{2}$$

where σ_1 and σ_2 are the principal Cauchy true stresses in axial and circumferential direction respectively. The radius r and the thickness h of the membrane are with respect to the deformed configuration. The radial stretch λ_3 is determined from the incompressibility constraint

$$\lambda_1 \lambda_2 \lambda_3 = 1 \tag{3}$$

Then

$$h = \frac{H}{\lambda_1 \lambda_2}; \quad r = \lambda_2 R \quad and \quad r_i = r - \frac{h}{2} = \lambda_2 R - \frac{H}{2\lambda_1 \lambda_2} \tag{4}$$

where R and H are the radius and the thickness in the undeformed configuration. We may derive the two principal Cauchy stresses in the two-dimensional case for the infinitesimally long cylinder by neglecting $\sigma_3 = -p$ (p is inner pressure) approximately as:

$$\sigma_1 = \lambda_1 \frac{\partial \Psi(\lambda_1, \lambda_2)}{\partial \lambda_1}, \qquad \sigma_2 = \lambda_2 \frac{\partial \Psi(\lambda_1, \lambda_2)}{\partial \lambda_2} \tag{5}$$

By substituting these results to the equilibrium equations we find:

$$\begin{aligned} &\lambda_1 \frac{\partial \Psi(\lambda_1, \lambda_2)}{\partial \lambda_1} - \frac{\lambda_1 (\lambda_2 R - \frac{H}{2\lambda_1 \lambda_2})^2}{2HR} p = 0 \\ &\lambda_2 \frac{\partial \Psi(\lambda_1, \lambda_2)}{\partial \lambda_2} - (\frac{\lambda_2^2 \lambda_1 R}{H} - \frac{1}{2}) p = 0 \end{aligned} \tag{6}$$

The experimentally determined values of λ_1 and λ_2 in several points of the central part of our cylindrical membrane were substituted into the equilibrium equations (6). The resulting overdetermined system of nonlinear equations for the parameters k_1 and k_2 was solved by the nonlinear least squares method. The determined values were k_1=54900 kPa and k_2=-16.9. The function of the Helmholtz energy potential for these parameters is convex.

3. Deformation of cylindrical orthotropic membrane

Following the reasoning in [4] we can determine the main geometric features of the inflated membrane. As shown in Figure 1, a thin cylindrical membrane of air-spring has the initial radius of mid-surface R, and length 2L. Its initial wall thickness H is assumed to be uniform. The cylindrical membrane is inflated by the internal pressure.

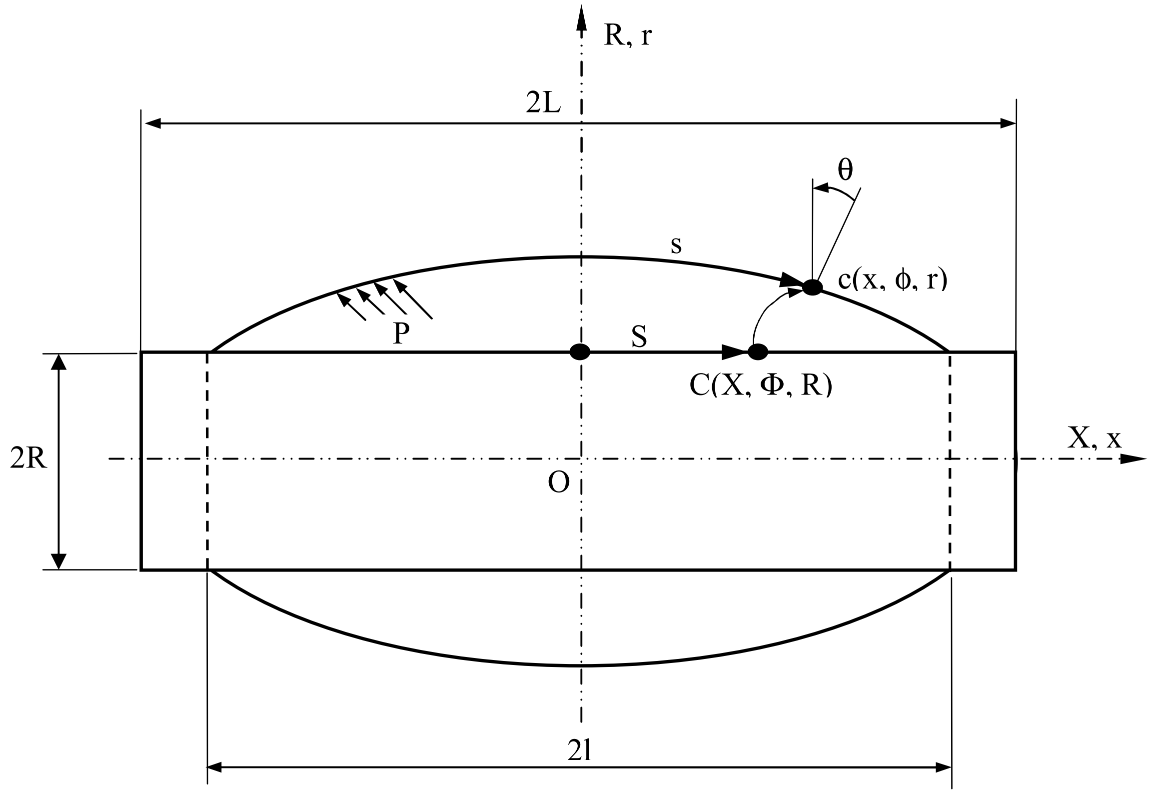

Figure 1

The deformed cylindrical membrane is referred to a cylindrical polar coordinate system (x, ϕ, r). A different cylindrical polar coordinate system, (X, Φ, R), is chosen for the description of the undeformed profile of membrane. A material particle moves during the deformation from the position in the undeformed profile, C(X, Φ, R) to the deformed profile, c(x, ϕ, r), along its quasi-equilibrium path. We assume the axisymmetric deformation, so that $\phi \equiv \Phi$. The principal stretch in axial direction, principal curvatures and geometric relations are

$$\lambda_1 = \frac{ds}{dS}, \quad \frac{dr}{ds} = -sin\,\theta, \quad \frac{dx}{ds} = cos\,\theta, \quad \kappa_1 = \frac{d\theta}{ds}, \quad \kappa_2 = \frac{cos\,\theta}{r} \tag{7}$$

where s is the arc length measured from pole (x = 0) to the particle c(x, ϕ, r) along the meridian of the deformed profile. S is the length corresponding to s in the undeformed profile. We introduce an auxiliary variable θ, the angle of the tangent line.

4. Governing equations

The theory of nonlinear membranes has been presented by Green and Adkins [8] and applied to various inflated structures [4]. The quasi-static equilibrium equations of problem are

$$\frac{d}{ds}\left(T_1 r\right) = T_2 \frac{dr}{ds}, \qquad \kappa_1 T_1 + \kappa_2 T_2 = p \tag{8}$$

where p is the inner pressure, T_1 and T_2 are the stress resultant forces per unit length of the meridional and circumferential directions. The stress resultant forces in the deformed configuration are

$$T_1 = h\,\sigma_1\,, \quad T_2 = h\,\sigma_2\,, \tag{9}$$

where Cauchy stresses σ_1 and σ_2 are given by (5). After the substitution of (4), (5), (7) and (9) into equations of equilibrium (8) we get after some simplifications the system of five ordinary differential equations with respect to the coordinate X of the undeformed configuration:

$$\begin{aligned}
&\frac{d\lambda_1}{dX} = \frac{\sin\theta}{R}\left(\frac{\partial^2 \Psi}{\partial \lambda_1^2}\right)^{-1}\left(\frac{\partial^2 \Psi}{\partial \lambda_1 \lambda_2}.\lambda_1 - \frac{\partial \Psi}{\partial \lambda_2}\right),\\
&\frac{d\theta}{dX} = \left(\frac{\partial \Psi}{\partial \lambda_1}\right)^{-1}\left(\frac{\lambda_1\lambda_2}{H}.p - \frac{\cos\theta}{R}\frac{\partial \Psi}{\partial \lambda_2}\right),\\
&\frac{d\lambda_2}{dX} = -\lambda_1 R^{-1}.\sin\theta\,,\\
&\frac{dx}{dX} = \lambda_1.\cos\theta\,,\\
&\frac{dp}{dX} = 0\,,
\end{aligned} \tag{10}$$

for the principal stretches λ_1 and λ_2, the tangent angle θ, the coordinate x in the deformed configuration and the inner pressure p. We substituted our strain energy function (1) into equations (10) and solved them numerically in Matlab with the boundary condition for λ_1^0 and λ_2^0 determined from the experiments. The results are at the Figure 2, where calculated stretches are compared with experimental ones.

5. Conclusions

The deformations of the nonlinear composite membrane were determined experimentally. The problem of the identification of the material parameters was solved. The proposed strain energy function was implemented into the calculus of deformation field of the cylindrical membrane. The deformations were determined by numerical solution the system of ordinary differential equations based on the membrane theory. The method will be used for the inverse identification of material parameters of the inflatable structures namely air-springs.

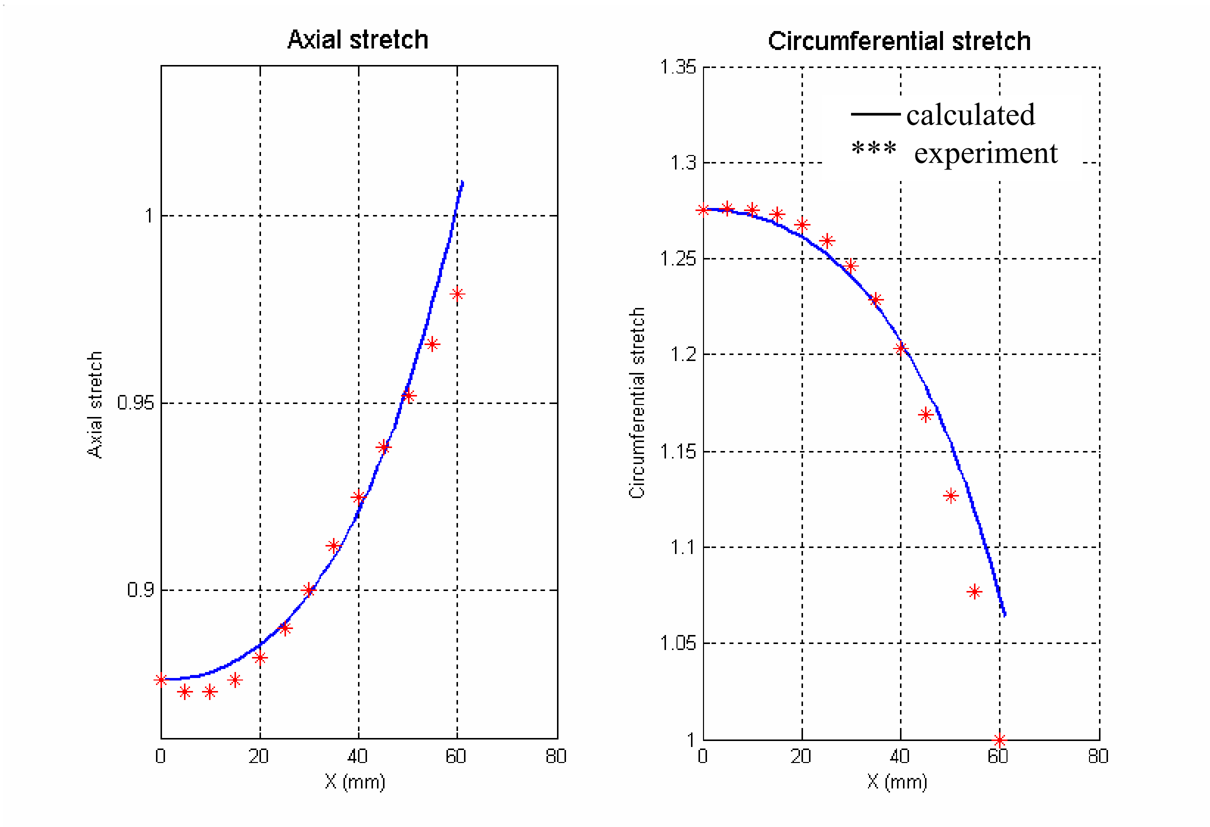

Figure 2

Acknowledgement

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